Voids Beneath Slabs-on-Ground

Using the impulse-response test to verify adequate slab support

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A concrete slab-on-ground is one of the most popular of all floor types for many applications, especially research facilities, where they usually provide the highest quality with respect to floor vibration. They are stiff and uniformly supported, so they don't exhibit the resonance characteristics of a suspended slab. As a result, they are much less responsive to footfall.

Occasionally, the value of a slab-on-ground is degraded because the slab separates from the subgrade, leaving a void. If this void has a large plan area, the floor will behave more like a suspended floor, exhibiting resonance and becoming more responsive to footfall and mechanical excitation. In a laboratory space, this could render the floor unfit for the lab equipment in the area. Separation from the soil can also weaken the floor and lead to cracking.

It's often desirable to detect a void or determine its horizontal extent, as well as verify that corrective measures have been successful. The results of one such remedial study, with measurements before and after remediation by grout injection, are presented in this article.

DYNAMIC CHARACTERISTICS OF A SLAB-ON-GROUND

A slab-on-ground may be considered a plate on an elastic foundation, assuming that the soil is elastic. When a vertical point load is applied to the center of a slab, it will deform downward in a dish shape with the maximum deflection occurring beneath the load. The local reaction pressure of the soil against the underside of the slab will be a function of the deflection in that vicinity. It has been shown¹ that the reaction pressure *p* of an elastic foundation beneath a plate may be given as

$$p = Sw = \frac{wD}{l^4} \tag{1}$$

where S is the modulus of subgrade reaction of the soil

beneath the slab, w is the local displacement downward, and l is the characteristic length. D is the plate rigidity, defined by

$$D = \frac{Et^3}{12(1 - v^2)}$$
(2)

where *t* is the slab thickness, and *E* and v are Young's modulus of elasticity and Poisson's ratio for the concrete, respectively. The term *l* in Eq. (1) is defined as

$$l = \sqrt[4]{D/S} \tag{3}$$

These relationships may be arranged to define a point stiffness k_s in terms of the slab thickness and elastic properties of the concrete along with the subgrade modulus written as

$$k_s = 8t^{1.5} \sqrt{\frac{SE}{12(1-\nu^2)}}$$
(4)

The subscript *s* on the stiffness indicates that it's a static stiffness. As with a single degree of freedom system, the point resistance of a slab-on-ground varies with the frequency of the applied load, defining the notion of dynamic stiffness, or stiffness that varies with frequency. Generally, dynamic stiffness is identical to static stiffness at frequencies significantly below the fundamental resonance frequency of the slab-foundation system.

We've observed that a slab that is in solid, uniform contact with the subgrade does not usually exhibit resonance at frequencies less than about 80 Hz, and that the stiffness is relatively constant up to this frequency. A void of significant horizontal dimension, however, will result in resonance at a frequency less than about 80 Hz that is observable using certain types of dynamic measurement techniques.



Fig. 1: During an impulse response test, the computer measures the signals produced by a load cell in the hammer and a motion sensor



Fig. 2: A typical slab-on-ground receptance spectrum with a peak at 80 Hz that indicates a resonance frequency (1 m/N = 175 in./lb)

In the sections that follow, we'll illustrate the dynamic characteristics of both a well-supported slab and a slab over a void and discuss how that void was remediated. Follow-up measurements demonstrate that the remediation was successful, eliminating resonance and producing behavior similar to the well-supported portion of the slab.

MEASURING IN-PLACE SLAB PROPERTIES

The impulse-response test^{2,3} permits measurement of frequency-dependent response to an impact load. Test results are presented in a manner representing the dynamic response (motion amplitude per unit force) or the dynamic resistance (force amplitude required to cause a unit amplitude of motion).

Response to the impact load may be represented in terms of displacement, velocity, or acceleration spectra,



Fig. 3: The dynamic stiffness spectrum for the measurements shown in Fig. 2 is the inverse of the receptance spectrum. The dashed line is the static stiffness of the slab-foundation system. The resonance at 80 Hz now appears as a dip (1 N/m = 0.00571 lb/in.)

each divided by the excitation force spectrum, in which case the resulting normalized spectra are known as receptance (or compliance), mobility, or accelerance, respectively. The inverse of receptance is dynamic stiffness. For greater detail on the representation of impulse-response data, see Reference 2.

The impulse response of a point on a structure is straightforward to measure with an impact force source. a vibration sensor, and a two-channel spectrum analyzer.² As shown in Fig. 1, a hammer equipped with a force sensor may be used to apply an impact force and measure the force created. An accelerometer, a sensor that measures acceleration, may be used to measure the dynamic response to the impact, and the signal may be double-integrated electronically to create a displacement signal. The spectrum analyzer can sample the force and response signals simultaneously and produce a receptance spectrum, such as the one shown in Fig. 2. The receptance spectrum shows the receptance as a function of frequency. The dynamic stiffness spectrum, shown in Fig. 3, is the inverse of the receptance spectrum. The static stiffness $k_{\rm s}$ at the point of load application is the asymptotic value of dynamic stiffness at low frequencies. This is shown as the dashed line in Fig. 3.

The presence of a void of significant size is best identified by a deviation of the spectrum from "normal" for a given slab, which can be defined based on the statistical range of the dynamic properties. Equation (4) shows that the point stiffness depends on the slab thickness, the elastic properties of the concrete, and the subgrade modulus, all of which may vary over a large area. To account for this expected variation of dynamic response for good support conditions, receptance may



Fig. 4: Receptance spectra measured at 22 locations on a 150 mm (6 in.) thick slab-on-ground (1 m/N = 175 in./lb)

be measured at a large number of locations, as shown in Fig. 4, which represents measurements at 22 locations. There are some peaks that represent resonances, but they all lie at frequencies above 70 Hz. The shapes of the spectra below that frequency are virtually identical. The variation in response is summarized in Fig. 5 in terms of the logarithmic mean (heavy curve) and the logarithmic mean \pm 1 standard deviation of the logarithmic spectra (light curves). The logarithmic mean at a given frequency is the mean of the logarithms of the value at that frequency for all of the individual spectra. The standard deviation of the logarithmic spectra is obtained in a similar manner. The range represented by the light curves in Fig. 5 will be called the "one-sigma limit." The average coefficient of variation of the data at frequencies less than 70 Hz is 52%.

Though there are exceptions, the spectrum of log mean receptance exhibits a characteristic we find typical of slabs-on-ground: it may be approximated by two straight lines—one horizontal and the other sloping. In this case, they intersect at about 80 Hz.

SLABS WITH UNDERLYING VOIDS

Several locations on the slab were suspected of curling at the edges, causing the slab to lift off the subgrade. The spectra in these areas were not included in the set shown in Fig. 4 representing "normal" behavior. Several measurements from one of the curled areas are shown in Fig. 6 and are compared with the one-sigma limits of the data measured at locations with normal bearing. Point 1 was near the softest location of the slab, in this case, near a corner. Point 3 was much closer to where the slab contacted the soil. Several characteristics of the curves for Points 1 to 3 may be observed by comparing them



Fig. 5: The log mean receptance for the 22 locations shown in Fig. 4 and log mean $\pm 1 \log$ standard deviation or one-sigma limits (1 m/N = 175 in./lb)



Fig. 6: Receptance spectra measured at three locations over a void, compared with the log mean and one-sigma limits of a normally supported slab (1 m/N = 175 in./lb)

with the curves representing normal behavior:

- The receptance spectra over the voids exhibit a peak at the same frequency, representing the fundamental resonance frequency of the unsupported slab. In this case, the frequency is about 30 Hz;
- The portion to the left of the resonance frequency, representing the inverse of static stiffness, is significantly higher than the one-sigma limits of receptance for this slab. In this case, the receptance at Point 1 is about eight times the average for the supported slab; and
- The slope of the segment to the right of the resonance peak differs noticeably from the slope of the log mean receptance for the data measured at locations with normal bearing.

It may be adequate to simply confirm the presence or absence of voids. In the event, however, that it's necessary to document the lateral extent of a void, the receptance spectra can be obtained on a grid pattern. A low-frequency resonance will be present at all locations over the void, and it will disappear when the slab comes in contact with the subgrade. It's impossible to determine the height of the void from these measurements. This requires coring and direct measurement.

VOID REPAIR

A void below a slab may be filled using grout injection, although significant care is required. If the grout is over-pressurized, the slab can lift, causing voids at other locations. When properly implemented, however, the slab is once again brought into solid contact with the subgrade.

The void at the test location represented in Fig. 6 was repaired using grout injection by an experienced geotechnical contractor. A collection of 50 mm (2 in.) diameter holes were drilled through the slab at 21 locations along the curled edge. The holes were spaced about 600 mm (24 in.) apart along a line about 600 mm (24 in.) from the edge. At every other hole, another hole was drilled about 300 mm (12 in.) from the edge. The void space at each hole was measured, and the height ranged from 3 to 25 mm (1/8 to 1 in.).

About 0.1 m³ (0.13 yd³) of fly ash and portland cement grout was pumped into the void via the cored holes. The grout was tested on site to have a 14-second efflux time flowing through a standard flow cone in accordance with ASTM C939.⁴ When the pressurized grout traveled to adjacent holes and returned up toward the surface, those holes were plugged. This allowed the grout to migrate to other holes. All holes were patched with a nonshrink, high-strength grout recommended by the contractor.

BEHAVIOR WITH A REPAIRED VOID

After the grout cured, impulse-response tests were performed once again to verify the remediation. The locations that had the abnormal receptance spectra were tested again. Figure 7 shows the test setup at the area associated with the data in Fig. 6. Figure 8 shows the receptance spectra before and after the repair at the location identified as Point 1 in Fig. 6. The repair led to a measured receptance that was within the one-sigma limits for a normally supported slab. In addition, tests were performed at locations beyond the perimeter of the void to ensure there had been no lifting due to excessive grout pressure. None was indicated.

After the slab remediation, the construction was completed. As part of the building commissioning process, a vibration survey was carried out in the laboratory wing of the building. As is customary, the vibration data in the operating facility were characterized statistically using the log mean and one-sigma limits of the survey data.⁵ Figure 9 shows the post-construction ambient velocity spectrum at the location identified as



Fig. 7: Test setup at repaired area. Arrows indicate the plugged injection sites



Fig. 8: Measured receptance spectra at the softest location before and after grout injection compared with the log mean and one-sigma limits for response of a normally supported slab (1 m/N = 175 in./lb)

Point 1 in Fig. 6, along with the log mean and one-sigma limits. The repair led to a measured ambient velocity spectrum that was within the one-sigma limits for a normally supported slab. The sharp peaks represent the vibrations from rotating mechanical equipment, and their amplitudes are not problematic.

Measurements were also made using footfall excitation, as this was more likely than ambient conditions to excite floor resonance, if it occurred. Figure 10 shows the velocity spectrum at the same location, along with the log mean and one-sigma limits for footfall at all of the measured locations. There are no resonance peaks evident in the data, and the measured footfall velocity spectrum was within the one-sigma limits for a normally supported slab.

References

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Fig. 9: Ambient velocity spectrum (heavy line) at the softest location after grout injection measured with the facility in operation. The log mean (light, solid line) and one-sigma limits (light, dashed lines) for ambient vibration of a normally supported slab are shown for comparison (1 μ in./second = 25.4 nm/second)



Fig. 10: Footfall velocity spectrum (heavy line) at the softest location after grout injection. The log mean (light, solid line) and one-sigma limits (light, dashed lines) for ambient vibration of a normally supported slab are shown for comparison (1 μ in./ second = 25.4 nm/second)

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Selected for reader interest by the editors after independent expert evaluation and recommendation.



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