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A Review of Several Methods for Processing Vibration Data

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ABSTRACT

For many areas of acoustics, standards organizations or regulatory bodies have mandated vibration or noise criteria and defined the appropriate processing methods. No such standards exist for vibration-sensitive facilities at this time except as defined by equipment manufacturers, facility owners, and/or vibration consultants. The existing criteria from these groups differ widely in form. This paper reviews the various candidate methods offered by current technology for processing measured vibration data.

1. INTRODUCTION

Current signal-processing technology offers a wide variety of methods for analysis of vibration measurements. A few of these have become conventional practice among those dealing with vibration control in vibration sensitive facilities, but there are still significant differences between the forms of criteria and measurement methods in common usage. [A proposed Recommended Practice document from the Institute of Environmental Sciences sets documentation requirements and limits the forms that should be used for vibration analysis but does not define criteria.]

Vibrations may be analyzed in terms of velocity, displacement, or acceleration; and all three metrics are commonly encountered by the analyst. One may evaluate vibration time histories or use their spectrum representation. Frequency bandwidth must be considered, addressing both the nature of the vibrations themselves (broadband or tonal) and that of the signal processing method (proportional or constant bandwidth). Both spectral and time-history data may be stated in terms of energy-average, peak-hold, and statistical centile quantities. Power spectrum density is finding increasing favor in some circles.

This paper reviews the various candidate methods offered by current technology for processing measured vibration data. Its intent is not to propose particular criteria or a single method for analysis, but to provide a means for comparing the different methods and the results obtained through their use. To facilitate this comparison, data are shown that have been processed by several different methods.

2. TIME-HISTORY VS. SPECTRUM REPRESENTATION

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The amplitude of vibratory motion can be expressed in terms of displacement, velocity, or acceleration. Vibrations can be measured using either velocity or acceleration sensors; electronic circuits can be used to convert from either of these metrics to one of the others. The output from a typical sensor is a time-varying analog signal. Figure 1 shows a representative 10-sec sample of velocity time history measured in the vertical direction on the floor of a 10-year-old operating fab. The peak velocity is about 1100 microinch/sec.

Typically, peak velocity taken from a time history is not very useful for assessing effects of vibrations on sensitive equipment. It provides no information regarding frequency content, and individual peaks contain relatively little energy with which to excite the internal resonances of equipment. Figure 2 shows the same time history upon which two slightly more useful history-based metrics have been superimposed. The lower straight line represents the energy average (root mean square or rms) amplitude of 350 microinch/sec resulting from a 10-sec average. The upper wavy line represents an energy average resulting from a 1/2-sec moving average, which has a maximum value of 485 microinch/sec. These two metrics are a little more meaningful, since they represent energy averages giving a suggestions of how equipment might respond; but they do not provide any frequency information.

Figure 3 shows the constant bandwidth velocity spectrum (400 lines, 0.375 Hz bandwidth, Hanning window) of a 3-min energy-averaged sample of the motion shown in Figs. 1 and 2. The numbers identify particular features of the vibrations at this location (discussed in Table 1), none of which is obvious from the time history alone. The predominant vibration is a tone at 59 Hz, which is probably a combined result of one or more items of rotating equipment with a shaft rate near 3540 rpm and the x2 harmonic of reciprocating equipment (vacuum pumps) with a shaft rate of 1765 rpm (see tones marked 4, 8, and 10 that are x1, x2, and x3, respectively).

The use of a frequency spectrum to represent measured vibration data offers the following advantages over a time history.

- It allows actual amplitudes to be correlated with known frequency-dependent sensitivities of equipment.
- It averages the time history over an integration time period, thus removing from consideration the "instantaneous' peak with little energy, which may have no effect on sensitive equipment.
- It excludes from consideration the frequency components that are of little consequence (in this case, those over 100 Hz).
- It allows some identification of the nature of the source.

Table 1. Vibration components in Figure 3.

Component No.	Component
1	Recirculation air unit at 500 rpm
2	Reciprocating or loose equipment at 1035 rpm
3	Rotating equipment at 1179

Component No.	Component
4	Rotating equipment at 1770 rpm
5	x2 harmonic of 2
6	Process equipment at 2325 rpm
7	x3 harmonic of 2
8	Rotating equipment at 3550 rpm; x2 harmonic of 2
9	second plate resonance of floor
10	x3 harmonic of 4

3. BANDWIDTH CONSIDERATIONS

Random or broadband vibrations are those that cannot be precisely predicted as a function of time, although they may be transformed into or defined by a spectrum that has no sharp peaks. They are typically produced by such things as vehicle traffic, construction, and turbulence in piping and ducting. Constant frequency or tonal vibrations are those associated with a particular frequency, thus representing definable functions of time. They appear as sharp peaks on a spectrum and are generally associated with rotating mechanical equipment.

Each point on a vibration spectrum might be said to represent an amplitude that passes a frequency band. It might be produced by actual filtering or by Fast Fourier Transform (FFT). The spacing between the upper and lower frequencies of that band are called the *bandwidth*, and it can either be constant and independent of frequency (constant bandwidth analysis) or proportional to frequency (proportional bandwidth analysis). Constant bandwidth analysis is often called narrowband analysis and the bands termed "narrow bands."

The most common form of proportional bandwidth analysis uses one-third octave bands, for which bandwidth is 23 percent of the center frequency and three adjacent bands represent an octave, a doubling of frequency. This form of analysis may be carried out using analyzers based upon either parallel filters or FFI. Established standards define the conventional center frequencies. Bandwidth is defined for constant bandwidth analysis by the total frequency range to be analyzed, the number of bands (or "lines") in that range, and the type of "window" to be used. FFT analysis is the most practical means to produce these spectra, typified by Fig. 3.

Broadband vibrations that are relatively constant with frequency can be converted from one bandwidth to another using Eq. (1). Equation (2) can be used to convert from constant bandwidth to the one-third octave band with center frequency f_c.

$$V_{bandwidth2} = V_{bandwidth1}\sqrt{\frac{bandwidth2}{bandwidth1}} \tag{1}$$

$$V_{1/3octave} = V_{nb}\sqrt{\frac{0.23 f_c}{nb\_bandwidth}} \tag{2}$$

Equations (1) and (2) require an assumption that the vibrations in the bands being combined are broadband; they may be inappropriate when the larger band has both broadband and tonal components. Regardless of the mix of tonal and broadband vibration components, several energy-averaged smaller bands can be combined into one larger band by summing the energy in all of the component bands (i.e., taking the square root of the sum of the square of the amplitude of each band). This approach can be used to synthesize larger bands from smaller bands, but there is no way to extract smaller bands from large ones. (This should not be done with spectra resulting from maximum-hold analysis.)

Synthesis of a one-third-octave band with center frequency f_c from narrow bands is carried out using Eq. (3). Figure 4 illustrates how a one-third-octave band spectrum could be synthesized from the constant bandwidth spectrum of Fig. 3. (It should be noted that this construction is only an example; the actual synthesis carried out by an analyzer is far more complicated.)

$$V_{1/3octave} = \left[\sum_{f=0.89 f_c}^{f=1.12 f_c} V_{nb}^2 \right]^{1/2} \qquad (3)$$

It can be seen in Fig. 4 that the one-third octave bands dominated by tones (16 and 63 Hz) have levels only slightly higher than the tones; the other one-third octave bands made up of several tones and/or broadband are significantly higher than the narrowband levels.

4. Examining The Bandwidth Sensitivity Of A Typical Instrument

The vibration amplitude at which an operator of a 1000x microscope could just begin to detect the effects of the vibration was determined experimentally. The microscope was excited by both tones and broadband noise. The latter was filtered to pass one-third octave bands. At each frequency setting the amplitude was increased until image disturbance was just becoming visible and then the base vibrations energy-averaged. About 30 sec averaging time was adequate for tones; broadband vibrations were averaged for about 2 min.

In Fig. 5 the solid line represents the sensitivity threshold for side-to-side motion measured using tones. The solid data points represent energy-averaged amplitudes of filtered broadband vibrations. Except for the effects of resonances near 23 and 27 Hz, there is fairly good correspondence between the threshold determined by tonal and broadband vibrations. (It was our conclusion that the damping of these resonances was so low--and the resonance bandwidth so narrow--that the broadband vibrations did not have enough energy to excite these resonances except as represented by the dip in the 25 Hz band.) One might conclude that a vibration criterion of 1000 microinch/sec would be appropriate for either tonal or broadband vibrations except between 20 and 30 Hz, where one could not use the same criterion for both unless there were no tones present or one was willing to tolerate some perceptible jiggle. A lateral vibration criterion of 400 or 500 microinch/sec would be appropriate to prevent perceptible image degradation for both broadband and tonal vibrations at all frequencies between 10 and 100 Hz.

A number of vibration criteria, including one family of generic vibration criteria, has been expressed in terms of one-third octave band representation of vibration amplitudes. The original reasons for doing so were: [1,2]

A one-third octave band roughly corresponds to the half-power bandwidth of a system with 10 percent critical damping and may represent the way in which this system combines vibrations energy.

A one-third octave band spectrum represents a compromise between what might be said to be "too much" information (a narrowband spectrum) and "too little" information (overall peak or rms amplitude, which has no information about frequency) or "too little" detail (octave or larger bands).

5. The Meaningful Metric

Vibration amplitudes may be expressed as displacement, velocity, or acceleration--denoted d, v, or a--that are related to each other as defined in Eq. (4). A spectrum can be converted from one metric to the other by using Eq. (4) for each point in the spectrum.

$$a = (2\pi f)v = (2\pi f)^2 d \tag{4}$$

A spectrum is the best form to express a derived criterion, but is often convenient to try to reduce a criterion to a single value that can be met at all values of frequency, as was done above for the microscope. Several vibration consulting firms follow a practice of using single-valued velocity criteria because a constant value of velocity can be shown to form a lower-bound to many of the experimentally-derived manufacturers' criteria. [3,4]

6. Statistics of Processing

Most spectrum analyzers provide the capability to obtain energy-averaged, maximum-hold, and exponentially averaged spectra. An analyzer can be used with computer post-processing to develop statistical distribution of levels such that *centile spectra* can be obtained. A centile spectrum Ln is the envelope of spectral values that are exceeded n percent of the measurement period.

Figure 6 shows the Ll, L10, L50, and L90 spectra for measurements made of vertical vibrations of a fab's second-level production floor. The levels in each band were obtained by using parallel one-third octave band filters that were sampled at 1-sec intervals over the entire sample period of 45 min. A computer determined the statistical distribution in each band. The peak of each spectrum is in the 25 Hz band, corresponding to the vertical fundamental resonance frequency of the floor. The second peak in the 8 Hz band is due to an inadequately-isolated air-handling unit at 500 rpm. One can infer that the vibrations exceed 300 microinch/sec 1 percent of the time.

The statistical distribution of the levels in the 25 Hz band is shown in Fig. 7 using a probability axis. The essentially straight-line curve of the log of amplitude (or the level in decibels) vs. probability indicates that the data fit a "normal" distribution. The two extremes are the Lmax and Lmin levels plotted at the percent associated with one sample and all of the samples, respectively. With normal distribution, L50 and Leq or the energy-averaged level are essentially the same.

If the distribution was not symmetric (i.e., the data did *not* fall on a straight line) then L50 and energy average may not be identical. For example, if there were short-duration events producing much higher levels (such as is the case when one or two vehicles drive by a remote location during a long measurement period; then, the Lmax and Ln at small values of n are skewed upward. The energy average may increase to something like L20, but the L50 level might remain more or less unchanged.

7. Power Spectrum Density

An approach gaining favor in some circles is the use of power spectrum density (PSD) representation of vibration data. It might be said that it is more "absolute" in its representation of broadband data because the amplitude is normalized by dividing the amplitude by the square root of measurement bandwidth. In this manner, measurements of a broadband signal will give the same level regardless of whether one-third octave band or 0.375 Hz bands are used.

However, the PSD approach isn't ideally suited for representing tonal data. A single tone measured in one-third octave bands and divided by the square root of the measurement bandwidth will give a different result from when that same tone is measured using a bandwidth of 0.375 Hz. Furthermore, the PSD level of a tone in one proportional band will be different from that of a tone of the same amplitude occurring in a different band.

Figure 8 shows the PSD representations of the two spectra of Fig. 4. In bands dominated by broadband vibrations, the one-third octave band is something of a "fit." In the bands dominated by significant tones (most notably the 63 Hz band) there is poor agreement between the PSD of the tone and that of the proportional band. Figure 9 shows the PSD for the measured microscope sensitivity data shown in Fig. 5. Comparing Figs. 5 and 9, we can see much better correspondence between broadband and tonal sensitivity when using actual energy-averaged amplitudes than when using PSD. Although PSD may be an appropriate representation of broadband vibration in a form insensitive to bandwidth, Figs. 4 and 5 suggest that it is an appropriate form for stating criteria or measurement results when both broadband and tonal vibrations are involved.

8. Illustration: Comparing Criteria

It is a popular practice to develop generic vibration criteria for the design or evaluation of **a** facility. One objective of this practice is to represent the "common denominator" vibration needs of the various sensitive tools in the facility. As long as the criterion is adequately defined, there are few problems with this approach except on the occasion when one must work with facilities that have used several criteria and one must explain the differences to the client.

For purposes of illustration, let us compare two popular families of generic vibration criteria. One is stated in terms of velocity spectrum curves to be compared with data analyzed in constant bandwidth with 0.125 Hz resolution. The criterion from this family relevant to current-generation microelectronics, which we will denote "NB," is defined as 0.8 rms micron/sec at frequencies between 5 and 50 Hz. The other family is stated in terms of velocity spectra to be compared with data analyzed in one-third octave bands of frequency. The comparable criterion, which we will denote "TOB," is defined as 250 rms microinch/sec at frequencies between 8 and 100 Hz, decreasing from 500 microinch/sec at 4 Hz.

A random signal (broadband noise) that *exactly* meets criterion NB at all frequencies has a constant level at all frequencies when measured using constant bandwidth. The levels will change if bandwidth is changed. This same random signal will have different levels and a positive slope of 3 dB/octave when measured using proportional bands as required for criterion TOB.

Figure 10 compares criterion NB with TOB in terms of one-third octave bands, in which the TOB criteria are normally defined. The upper dashed curve represents a broadband signal that exactly meets the NB criterion in *all* of the narrow bands in each one-third octave band. Over the common frequency range of the two criteria (5 to 50 Hz) they are of approximately the same order of magnitude. The lowest dashed curve represents the amplitude of single-band or tonal vibrations that are acceptable to criterion NB. The other dashed curves represent signals that are non-tonal but have some percentage of the narrow bands exactly meeting criterion NB and the remainder at least 10 decibels lower. Criterion NB is much more stringent than TOB with regard to tonal vibrations.

Figure 11 compares the two criteria in constant bandwidth domain, in which the NB criteria are normally defined. The conversion of the TOB criterion to constant bandwidth requires assumptions about tonality or randomness of the vibration. If the vibration is a single tone, it can be of the amplitude shown by the upper TOB curve. If it is random and of amplitude that is uniform with frequency, it must lie below the lower curve. If not all bands are of the same amplitude, the allowable amplitude will be somewhere between the two TOB curves.

These comparisons point out the fact that the two forms are entirely dissimilar in their treatment of random and single-frequency vibrations. The NB criteria equally weight the random vibrations in 0.125 Hz bandwidth with single-frequency components. The TOB criteria, in effect, equally weight the total energy of random vibrations in any one-third octave band with the single-frequency components in that band. It is important that while generic comparisons may be drawn between the two criterion forms, comparison of field data with one or the other criterion must be made in the particular criterion's native domain.

9. Conclusions

A wide variety of processing methods is available to analyze vibration data. The one used for a particular application should be based upon judgment of the particular circumstances of that application. The type of measurement must be adequately documented to show the type of band and averaging, bandwidth and windowing, sample duration, and type of amplitude (energy-average, maximum-hold, or centile).

10. References

1. E. Ungar, personal communication, 1985.

2. C. G. Gordon, personal communication, 1987.

3. E. Ungar, D. H. Sturz, and H. Amick, 'Vibration Control Design of High Technology Facilities," *Sound and Vibration,* July 1990.

4. E. E. Ungar, "Designing Sensitive Equipment and Facilities," *Mechanical Engineering,* December 1985.

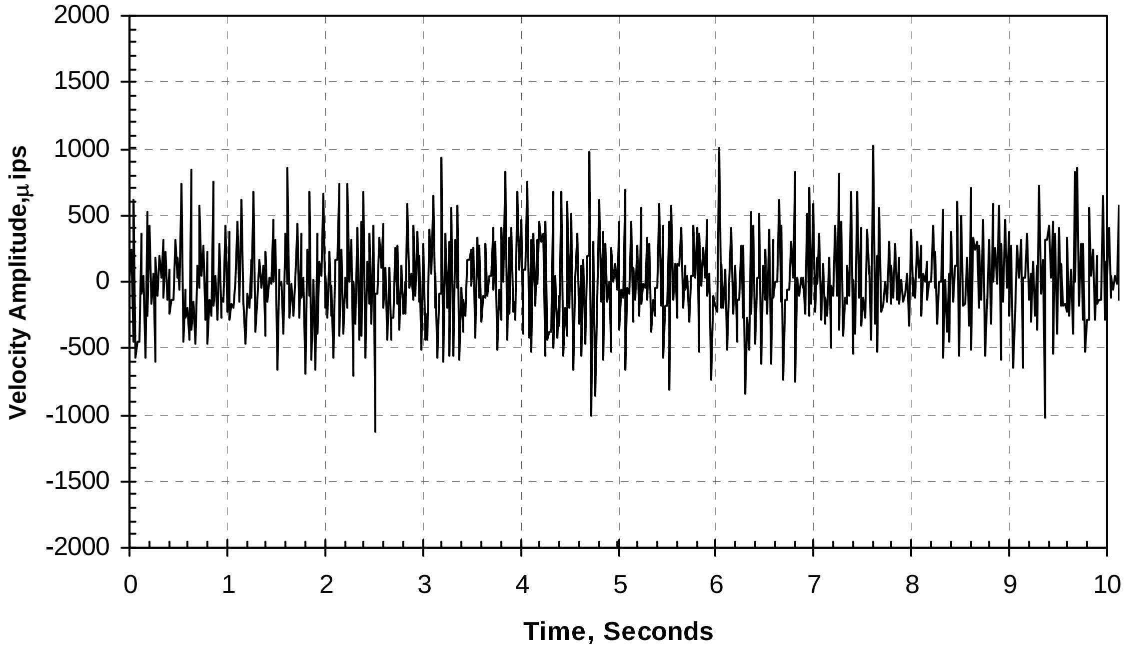

Figure 1. Representative 10-sec sample of velocity time history.

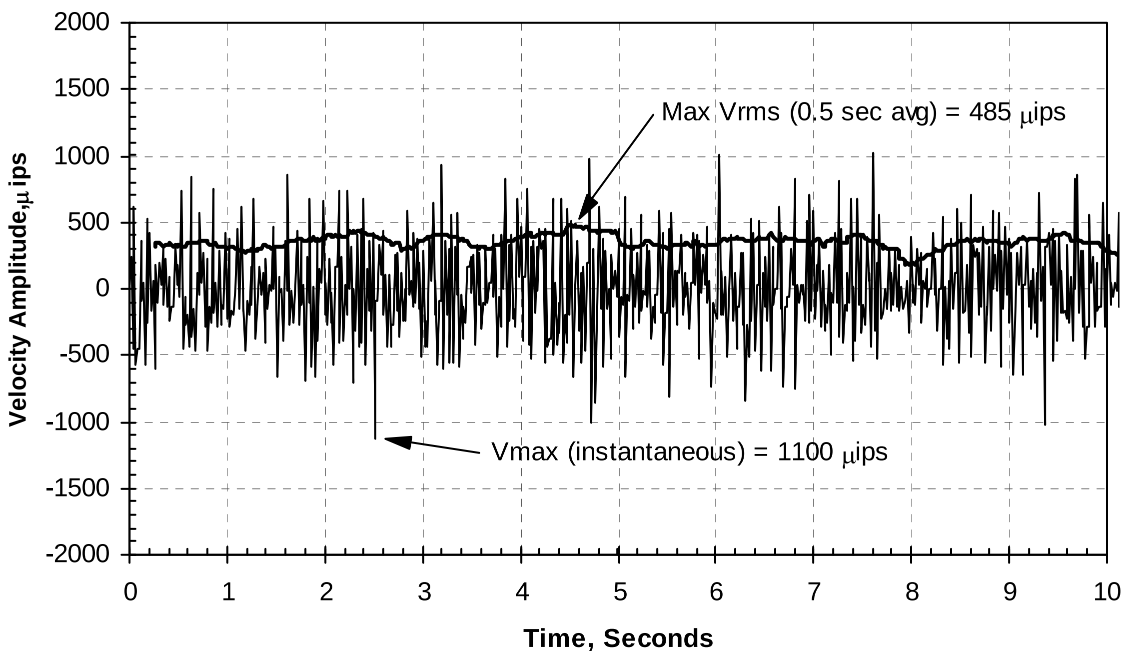

Figure 2. Representative 10-sec sample of velocity time history. Also shown are energy averages resulting from a 10-sec average and a one-half sec moving average.

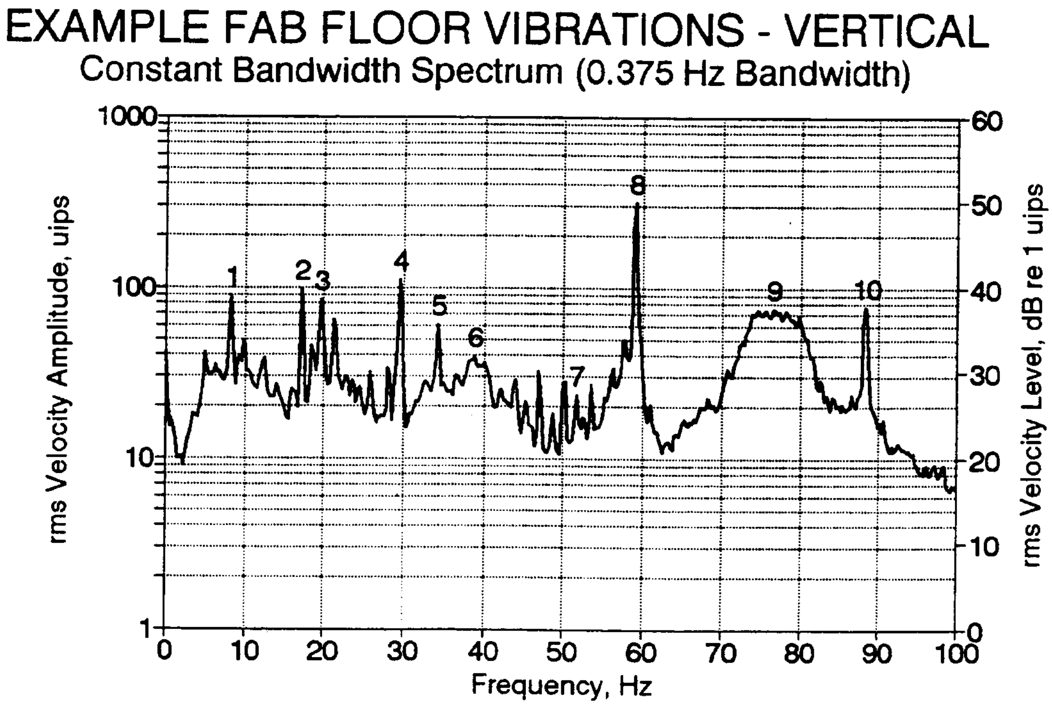

Figure 3. Constant bandwidth velocity spectrum of a 3-min energy-averaged sample of the motion shown in Figs. 1 and 2.

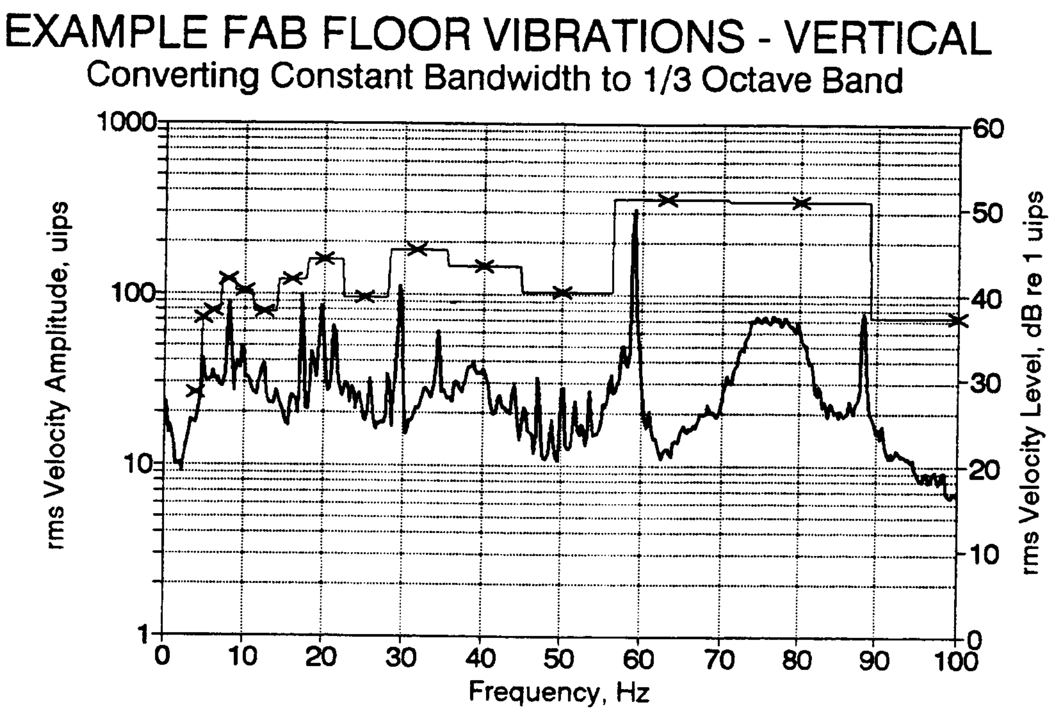

Figure 4. Synthesis of a one-third octave band spectrum from the constant bandwidth spectrum of Fig. 3.

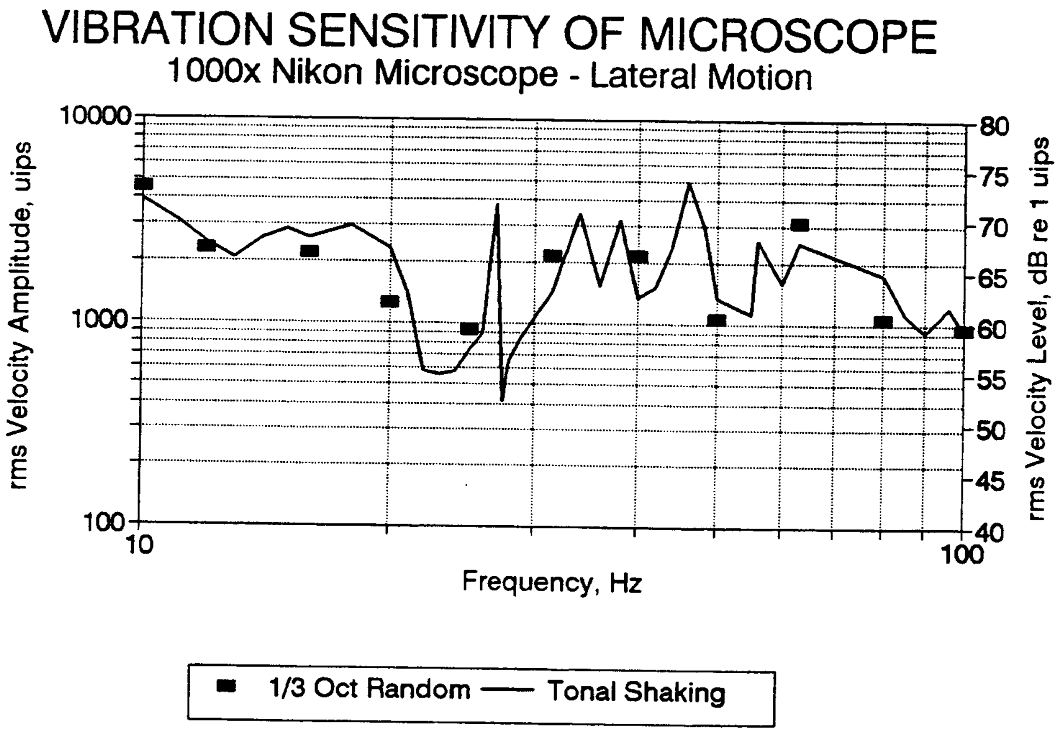

Figure 5. **Results of vibration sensitivity tests for optical microscope at 1000x, side-to-side motion.**

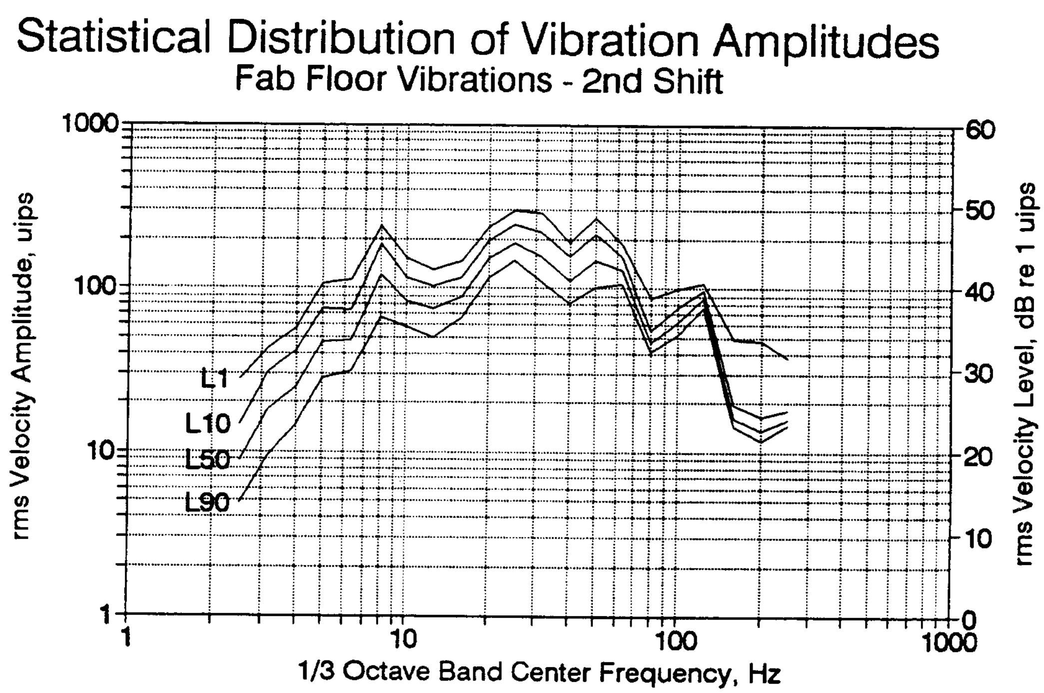

Figure 6. **Ll, L10, L50, and L90 spectra for measurements made of vertical vibrations of a fab's second-level production floor, obtained by using parallel one-third octave band filters.**

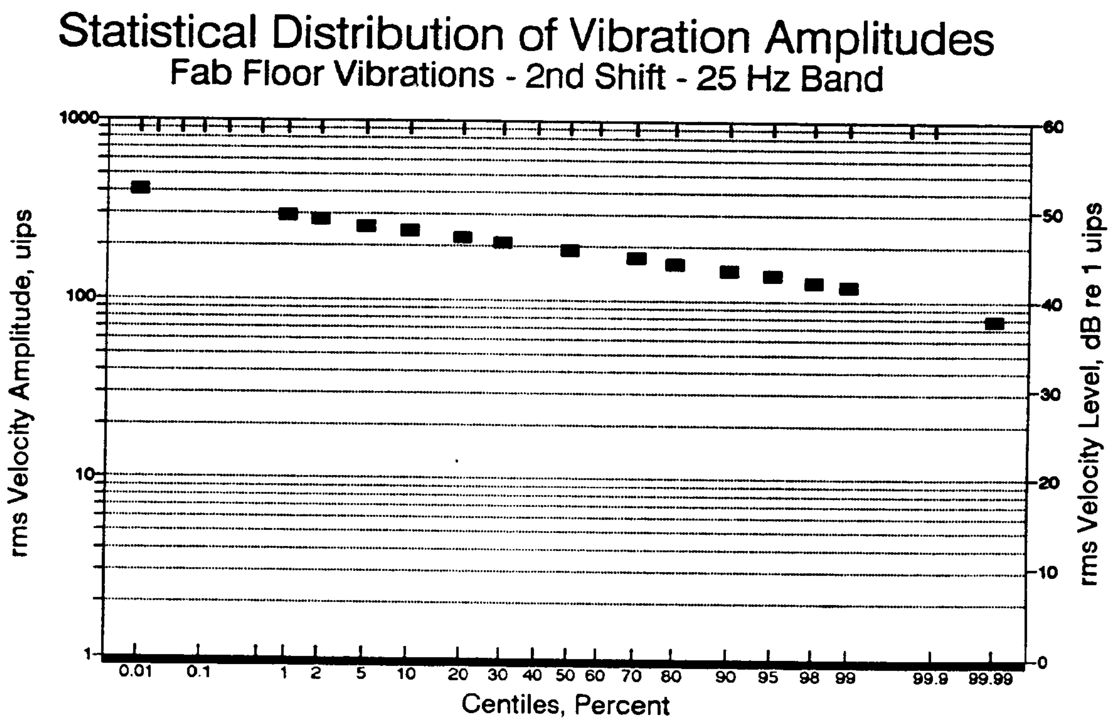

Figure 7. Statistical distribution of the levels in the 25 Hz band.

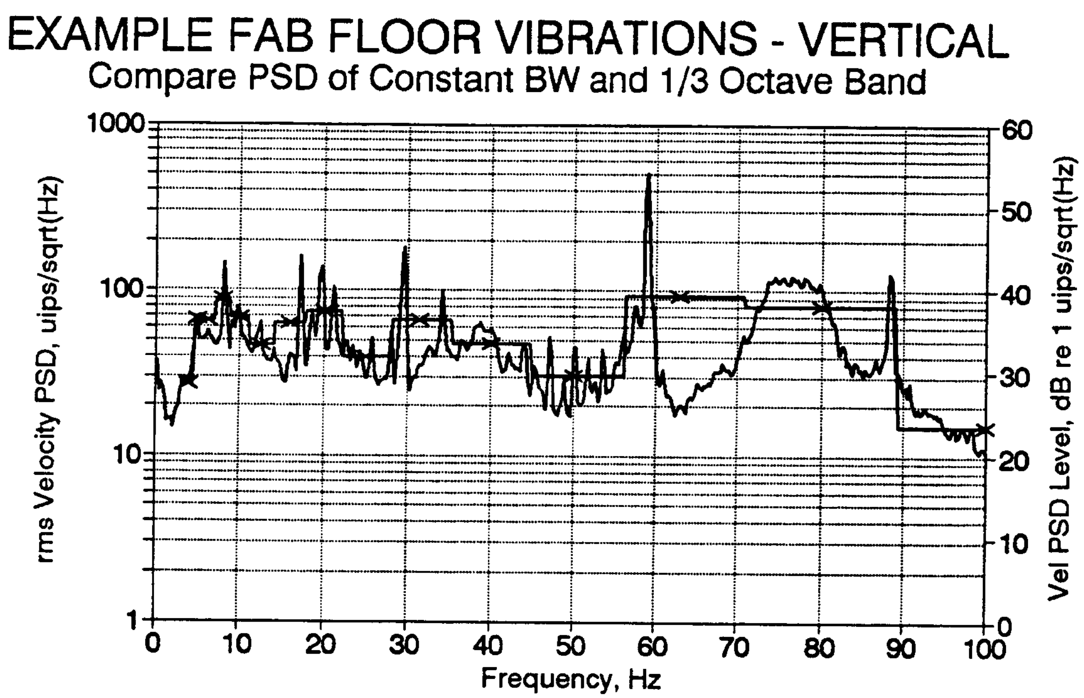

Figure 8. PSD representations of the two spectra of Fig. 4.

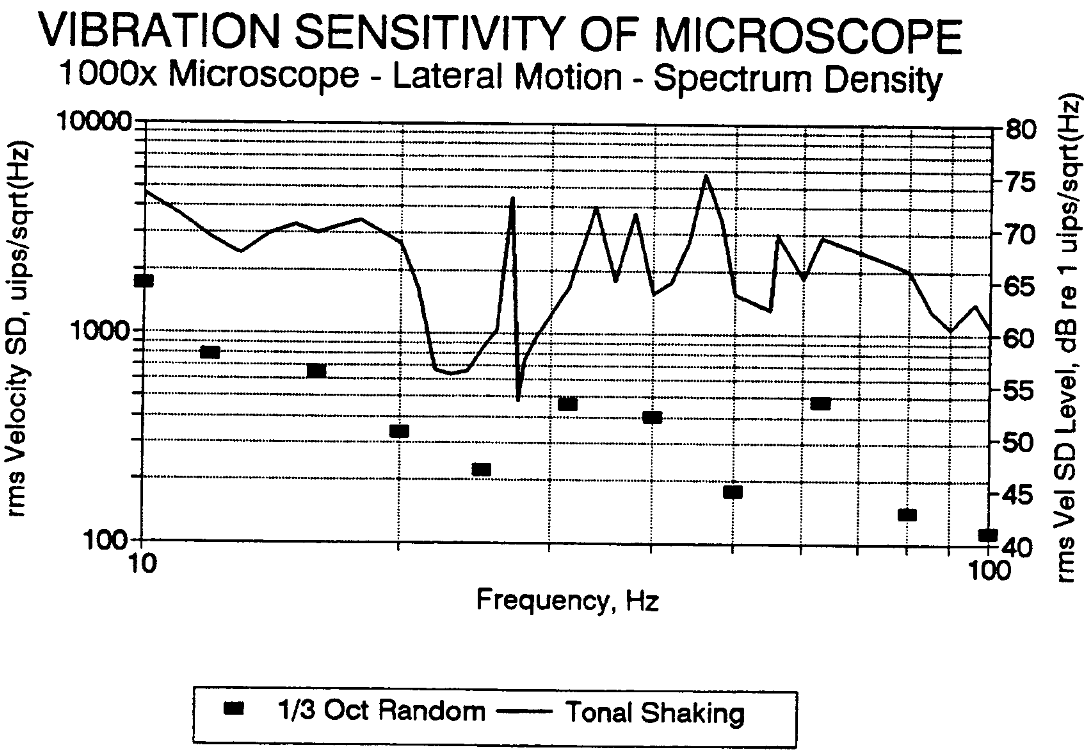

Figure 9. PSD for the measured microscope sensitivity data shown in Fig. 5.

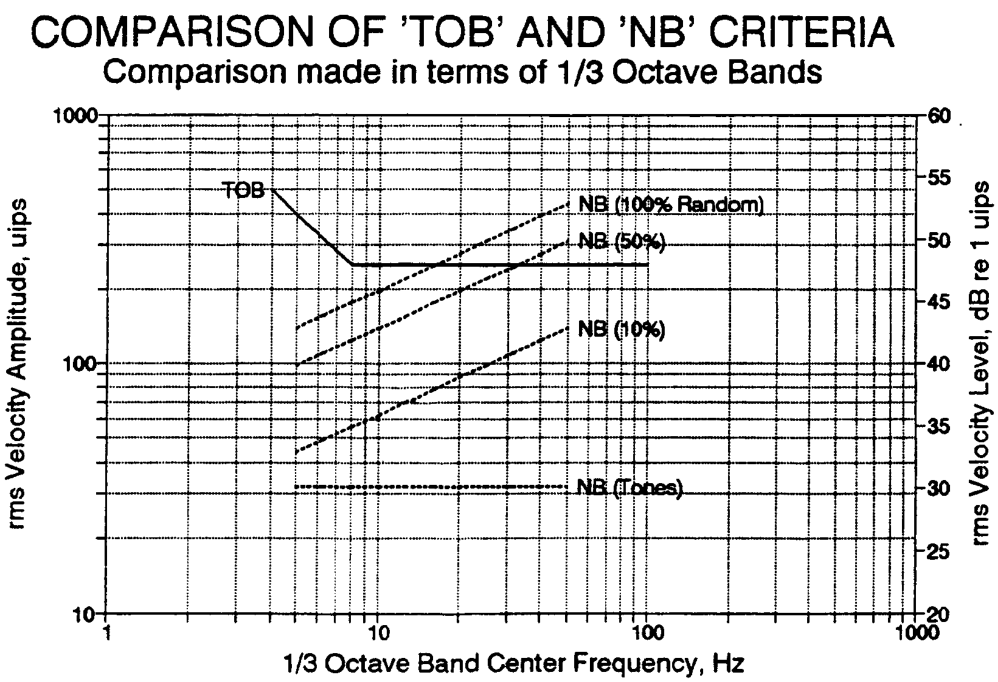

Figure 10. Comparison of criterion NB with TOB in terms of one-third octave bands. The upper dashed curve represents a broadband signal that exactly meets the NB criterion.

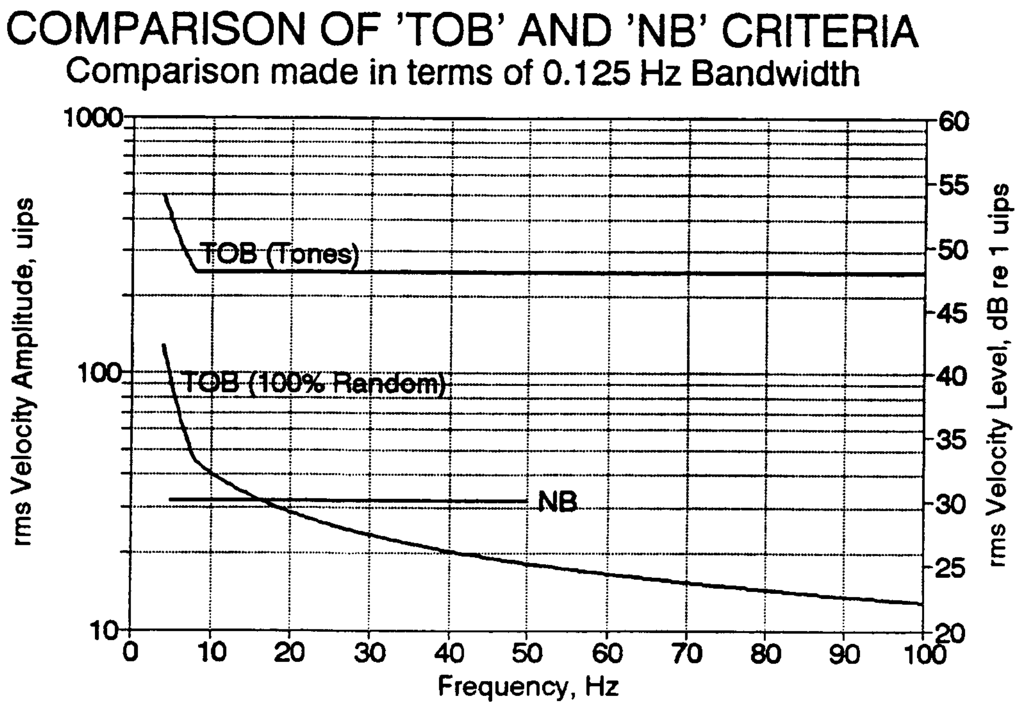

Figure 11. Comparison of criterion NB with TOB in constant bandwidth domain, in which the NB criteria are normally defined.