Mode selection for footfall analysis of floors

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ABSTRACT. Floor vibration due to footfall is of great importance in the design of buildings for vibration-sensitive research and production. The bulk of the predictive modeling methods for footfall involve the use of the "fundamental" resonance frequency which would be excited by the walker. In a complex structure, the analyst may be presented with a choice from a large collection of orthogonal modes which might be considered the "fundamental." Some may involve a large floor mass. Others may extend into adjacent structural bays. How does one realistically select the "best" one? The paper reviews the options typically available to the modeler and the approaches which might be used to select one for simple footfall analysis.

Introduction

Many situations require vibration analysis of suspended floors due to various sources. Vibration due to footfall is a common problem, and it is this source that is primarily discussed in this article. The transient- but repeated-nature of the forces applied by walkers requires special consideration in prediction of floor response.

In the prediction of footfall impact to floor structures, many common analysis methods are, among other parameters, expressed as a function of frequency [2,5,6,7]. Thus, identification of the fundamental vibration mode of the floor is necessary. The fundamental mode is often assumed to be the first (lowest frequency) mode. However, in practice, the fundamental mode, especially when the results are presented as velocity spectra as is the normal convention for many important design criteria, is not always the mode that is most significantly excited by transient impacts such as due to walkers, and assumption of the fundamental frequency would cause prediction errors. Mode extraction processes used in modeling of stiff floors may identify modes that are difficult or impossible to significantly excite by transient vibration, especially when the load amplitude is relatively low or non-continuous (e.g., due to a walker). This is sometimes also observed with relatively high amplitude transient loads, such as caused by large robotic machinery.

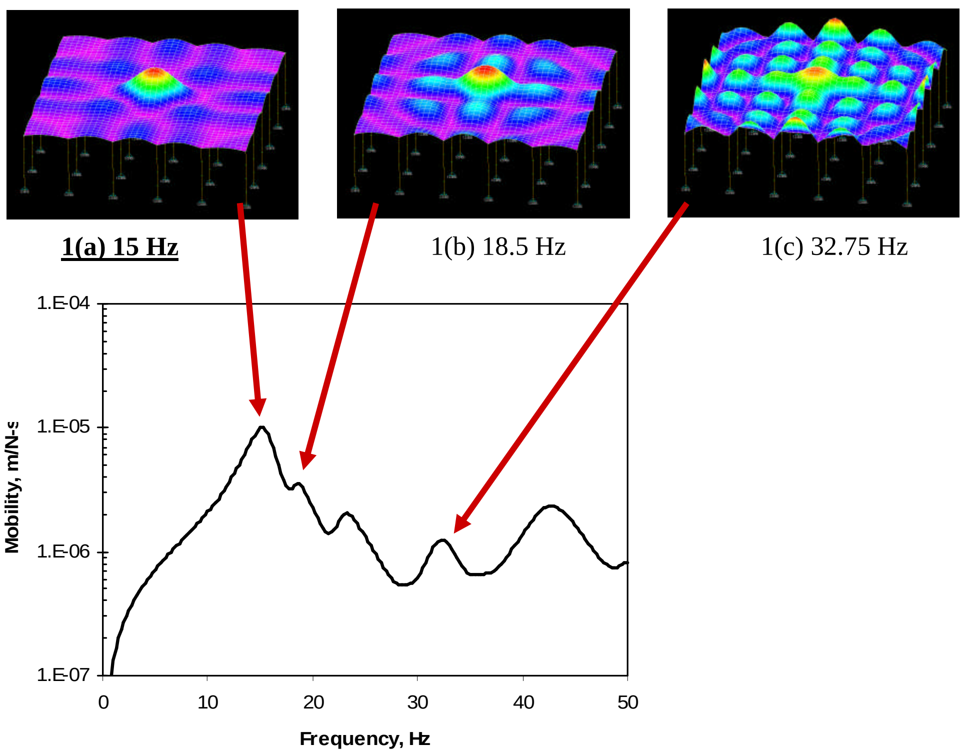
Due to the dependence on frequency, assumption of a modal frequency that would not be significantly excited by a walker produces inaccurate predictions. Since the referenced methodologies generally consider the frequency as inversely proportional to the resultant response (at least within their general range of applicability—this is a topic for another paper), the resultant designs tend to be overly conservative. While the opposite result would probably be worse (resulting in excessive vibration), unnecessary conservatism may result in an excessively costly design.

It is necessary to understand which particular mode shapes are most likely to be excited by transient walker forces. This sometimes requires considerations beyond results that may be obtained in dynamic analysis using finite element (FEA) methods. In this case, it is common to determine frequency response due to a swept sinusoidal load over a range of frequencies. However, FEA sinusoidal loading assumes long-term loading with time necessary to establish each mode. In contrast, footfall impulses are by their nature not continuous. Floor vibration modes that contain larger effective participating mass take longer to establish, and thus may not be highly excited by short-term impacts, even if repeated.

Definition of motion in suspended floors

By their nature, the response of floors that are suspended, whether by columns or trusses, is complex.[1] Specific frequency response in many cases cannot be described simply, except in floors of a most ideal configuration. Factors affecting the shape and frequency of the fundamental modes include aspect ratio (square bays[2] versus various rectangular shapes); the ratio of floor slab to column stiffness; dimensions of floor slabs (thickness and column spacing, etc.); material properties; any anomalous features such as penetrations or varying boundary conditions; etc. For example, Figures 1 through 3 each show three mode shapes[3] and a swept sine mobility spectrum for a 5 x 5 bay concrete floor slab, each with a slab thickness of 200mm, and without beams or girders (i.e., a "flat slab"). Three cases are illustrated: (1) a square bay arrangement with the floor supported on very stiff columns (such that they do not participate significantly in the first several modes) spaced at 5m x 5m; (2) the same floor slab on softer (participating) columns of the same spacing; and (3) a bay of rectangular configuration upon stiff columns (5m x 7.3m spacing).

Figure 1: 5m x 5m bays with "stiff" columns, mobility spectrum and illustration of three mode shapes



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[1] In this essay we are primarily concerned with the vertical response of floors. We are furthermore are primarily discussing floors "soft" enough to be excited by a walker; this includes floors designed for most moderately vibration-sensitive work (such as with optical microscopes, operating theatres, etc.) and those that are less sensitive such as sleeping areas and offices [4]. Much less common is another class of significantly stiffer floors, such as used for semiconductor production and the most sensitive metrology [1]. These respond differently to walkers, if at all, and are generally more impacted by other sources with much larger force profiles (mechanical equipment. etc.).

[2] A "bay" is the square or rectangle of floor bounded by four supporting columns.

[3] These are not "modeshapes" in the strict sense, but are actually representations of displacement magnitude associated with the indicated frequency. Unlike classical modeshapes, in which phase is preserved, the FEA software presents all amplitudes as positive, eliminating phase.

Figure 2: 5m x 5m bays with “soft” columns, mobility spectrum and illustration of three important mode shapes

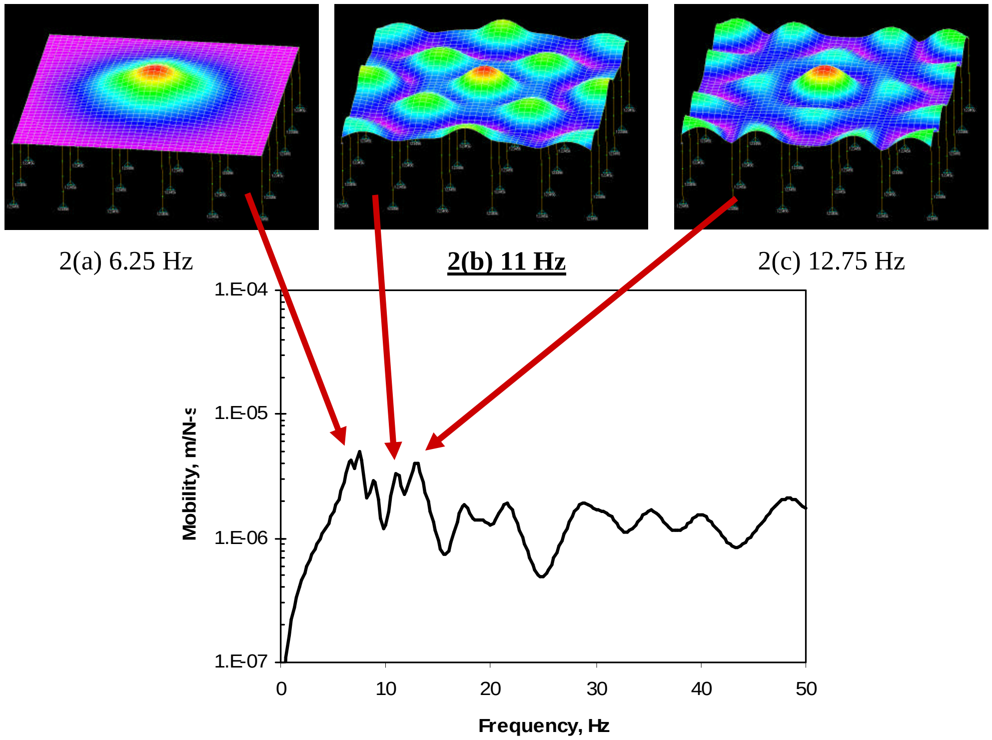
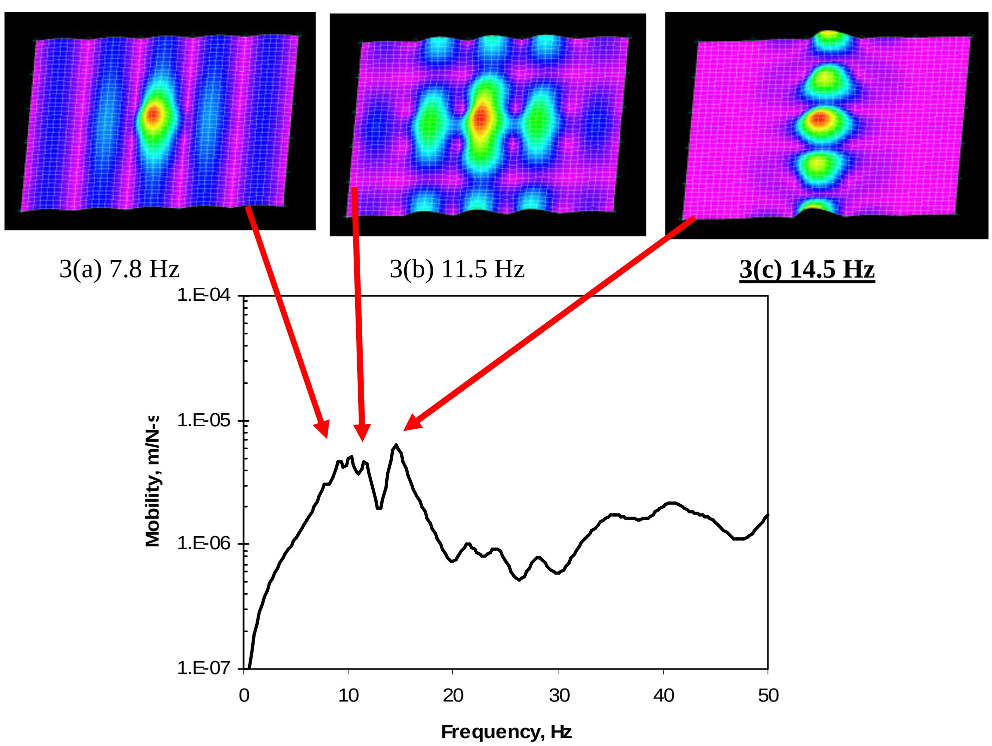


Figure 3: 5m x 7.3m bays with “stiff” columns, mobility spectrum and illustration of three important mode shapes



Several important mode shapes are shown in these figures. Modes 1a, 2b, and 3c show the first bending mode of the slab alone, without significant participation from the columns or adjacent bays (although the deformation may extend into adjacent bays to some degree).[4] This is sometimes referred to as the “oilcan” mode, similar to the motion the bottom panel of an antique oilcan necessary to pump oil. It is also similar to the first mode of another circular system with constrained edges, the circular membrane (a classic drum).

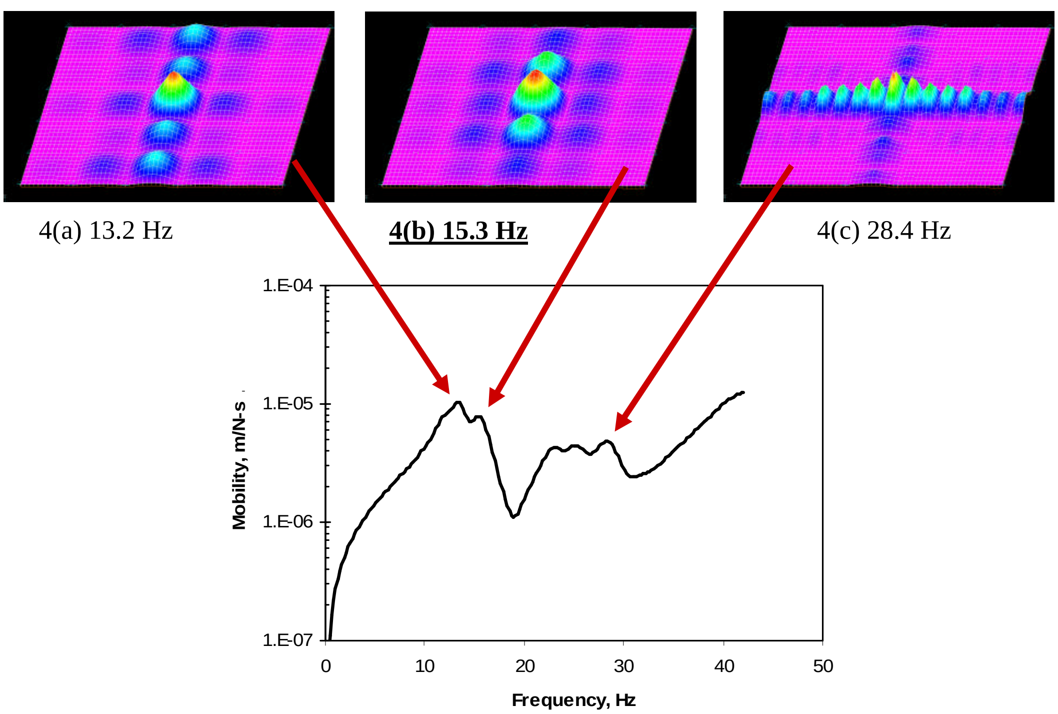
Mode 2a, and higher modes in Figures 1 and 3, are column modes: fundamental or higher order longitudinal modes of the columns that affect the vibration performance of the whole floor structure. Mode 3a and other modes lower in frequency than Mode 3c involve in-phase motion of several adjacent bays, and are primarily a characteristic of non-square bay configuration. These are sometimes referred to as “barrel” modes due to their cylindrical shape extending over several bays. These cases illustrate that in a column supported floor, the “fundamental” mode—or mode of lowest frequency—can take several different shapes depending on the floor configuration (oilcan, column, and barrel, respectively, in the cases shown).

As with any system with distributed properties, various modes may be excited simultaneously and superpose. The resultant amplitude of each of the excited modes is a function of where the load is applied, and the temporal and frequency content of the load, in particular.

Example: design and measurement on a real floor

The floor used in our example has the following configuration. The floor slab consists of 8.9cm thick concrete upon a 5.1cm steel deck, with W24x103 girders and four W24x68 beams per bay, set upon columns spaced at 6.7m x 9.1m. Figure 4 shows the calculated frequency response of the floor using FEA modeling. Modal frequencies at 13.2 Hz, 15.3 Hz, 28.4 Hz and others are indicated. The primary difference between 4(a) and 4(b) is the extent of mass being excited. The mode shown in 4(c) involves only the plate, with the steel beams forming (in essence) nodal lines of a bending wave propagating left and right.

Figure 4: Real floor, mobility spectrum and illustration of three mode shapes



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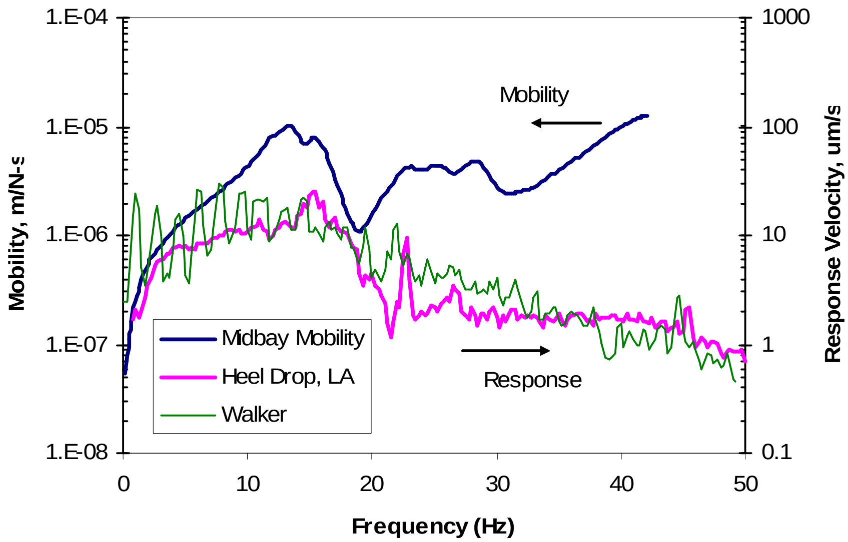
[4] Modes 2b and 3b are actually the fourth and fifth modes extracted in the analyses, respectively.

Figure 5 shows the resultant walker and "heel drop" vibration impact measurement data on the floor, along with the calculated mobility spectrum. The most significantly excited mode in the velocity spectrum for the heel drop is at 15.8 Hz, corresponding to the second mode of the floor (4b in Figure 4). (In the walker-generated spectrum, equally-spaced lower frequency peaks are also present, which are at integer multiples of the walker pace frequency.) Further analysis using the FEA model reveals that, as expected, this is the "oilcan" mode of the floor that is most easily excited, even though the floor contains lower frequency modes.

Figure 5: Real floor, with measured walker and heel drop vibration with primary impact (highest velocity) at 15.8 Hz, compared to calculated mobility spectrum



This example demonstrates why it is necessary to determine the mode that will be most strongly excited by a walker, and that this is not necessarily the fundamental mode. We can define the appropriate mode for analysis as the "critical" walker mode.

Walker impact and force spectrum

A walker force, by its very transient nature, varies in time [3]. The most basic and thorough analysis would use some means of applying the force in the time domain. However, it is also possible to represent walker impact as a spectrum in the frequency domain, with appropriate constants of translation to represent the quasi-static maximum impact. The latter representation is in many ways more convenient for analysis, and because most floor vibration criteria are expressed in the frequency domain.

Modal excitation by transients

Why are some modes well excited by a walker transient, and others not, even though they may share an antinode at the impact point (such as modes a, b and c in Figure 3)? Consider how the transient force interacts with the mobility spectrum of the object to which it is imparted. The most basic formulation of an undamped single degree of freedom oscillator, as used in some of the walker vibration calculation methods discussed earlier, is

$$F_f = F_r + M_p a, \text{ or } (F_f - F_r) = M_p a \qquad (1)$$

which sets the imparted force F_f against the restoring force of the floor (F_r) plus the participating mass of the floor (M_p) times acceleration. Rearrangement of this equation makes it clear that if $F_f << F_r$, F_f is negligible. In other words, if the restoring force, or, essentially, the participating mass of a particular mode, is much higher than that imparted by the walker force, the mode is not strongly excited.

Other effects likely play a role in emphasis of certain modes over others (again assuming coincident antinodes at the point of application of the force), include destructive interference of superposed modes, and damping, although these are probably secondary effects.

Calculation of effective modal mass

The degree to which a particular mode will be excited (or, more correctly, *established*) can be expressed as a ratio of the imparted force to the restoring force for the mode. Less fundamentally, because it does not account for variances due to walker pace, establishment may also be expressed as a ratio of the walker weight (mass) to the effective participating mass of the mode of interest. This assumes some method of determining the modal restoring force or effective participating mass at the design stage.

What we are calling *effective participating mass* for a particular mode is not the same as the *modal mass*, though it is similar to the *modal participation factor*.

Using the eigenvector modal frequencies[5] identified in the sweep tests such as shown in Figures 1 through 5, or from the corresponding receptance plots, we can estimate the effective participating modal mass according to Equation (2):

$$\hat{m}_i = \overline{m} \bullet \left|\overline{d}_i\right| \tag{2}$$

where $\overline{m}$ is the mass vector, including both dead load and effective live load; $\overline{d}_i$ is the normalized displacement vector (normalized by the amplitude at the load point) resulting from a point load and associated with the i^{th} frequency f_i; and $\hat{m}_i$ is the effective participating mass, a scalar quantity, associated with the i^{th} frequency f_i. In theory, then, one could compare $\hat{m}_i$ to the mass of the walker, or develop an expression of the ratio of modal restoring forces to the walker force, in order to determine the likelihood of excitation by this particular type of force.

Note also that similar results may be determined by inspection of modeled mode shapes, if these can be adequately visualized.

Conclusions

Predictive modeling of suspended floors for response from footfall is complicated, and appropriate considerations about modal response must be considered for accurate results. In particular, vibration modes with a high degree of effective participating mass may not be well excited by typical forces imparted by walkers, even though these may in some cases be the "fundamental" mode. However, consideration of the mode shapes and the ratio of forces (applied to restoring) may be used to select the appropriate mode for analysis.

References

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[4] International Standards Organization ISO 2631, *Guide to the Evaluation of Human Exposure to Vibration and Shock in Buildings (1 Hz to 80Hz)*, ISO 2631, International Standards Organization, Geneva, 1981.

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[5] The eigenvector modes are assumed to be orthogonal, and thus each may be treated individually as a single degree of freedom.

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